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SCALING LAWS FOR SATURATION INTENSITY AND LENGTH
IN AN UNTAPERED WIGGLER FREE-ELECTRON-LASER AMPLIFIER

by

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# SCALING LAWS FOR SATURATION INTENSITY AND LENGTH IN AN UNTAPERED WIGGLER FREE-ELECTRON-LASER AMPLIFIER[1]

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**Abstract:** A combination of dimensional analysis and numerical computation methods was used to derive scaling laws for saturation intensity and saturation length in a millimeter wave free-electron-laser amplifier with an untapered wiggler.[2]

**Key words:** saturation, scaling laws, free-electron laser (FEL)

## 1. Introduction

Free-electron laser fundamental mode radiation must pass through linear and non-linear stages to reach saturation. High saturation intensity $P_{SS}$ (the subscript SS means saturation value) is one of the most sought-after goals. Saturation intensity is related to linear stage growth rate (abbreviated to growth rate here). Research shows that when growth rate is high, $P_{SS}$ is not necessarily high, because $P_{SS}$ is related to both growth rate and saturation length $z_{SS}$ (the axial distance that fundamental mode radiation must travel to reach saturation). Under conditions approximating one dimension, the physical parameters that determine $P_{SS}$ and $z_{SS}$ of untapered free-electron-laser amplifiers are: incident electron beam current $I$, relativity factor $\gamma_0$, and relative energy dispersion $\Delta\gamma_0/\gamma_0$; wiggler magnetic field strength $B_w$ and period $\lambda_w$; fundamental mode laser wavelength $\lambda_s$ and original fundamental mode radiation power $P_{S0}$. Of these, $\gamma_0$, $B_w$, $\lambda_w$, and $\lambda_s$ satisfy a resonance relationship – see

[1] This is the English title given by the Chinese authors. A more literal translation of the Chinese title yields "Scaling laws for saturation power of amplifier-type free-electron lasers." Note that the original Chinese title says "amplifier-type FEL" rather than "FEL amplifier."

[2] *Changcanshu*, which the authors translate as "untapered," can also be translated as "invariable."

formula (7). Because the complete expression of the dependence of $P_{SS}$ and $z_{SS}$ on the above physical parameters is a non-linear problem, it is difficult to use total analysis methods on it. In addition, there are many physical parameters, so even if $\Delta\gamma_0/\gamma_0$ is given, there are also the four independent fundamental physical parameters $I$, $\gamma_0$, $B_w$, and $\gamma_w$. Thus, it is difficult to use a pure numerical fitting method to provide an expression.

Some newer, better methods that use a combination of dimensional analysis and numerical simulation are the United States' LLNL's[3] Garrison-Wong[1] (hereafter referred to as G-W) and Kumada-Sessler[2] methods. These proceed from the Liouville light field equation group, and use the characteristic intensity and characteristic length of R. Banifacio's dimensional analysis method to nondimensionalize groups of equations. The nondimensional saturation intensity and saturation length they obtain are two nondimensional parameters (which are combinations of the above four physical parameters) and functions of the nondimensional parameters that describe the original conditions (document [1] assumes that energy dispersion is zero); afterwards, under conditions where $\lambda_s = 1\mu m$ and $10\mu m$, they use numerical calculation to approximately determine these two functions. We used the G-W method to derive scaling laws for saturation intensity under millimeter wave conditions, but our results were not as good as those attained in document [1] under conditions where $\lambda_s = 1\mu m$ and $10\mu m$.

This article proceeds from a group of one-dimensional fundamental equations of single particle theory. The most appropriate characteristic intensity and characteristic length were chosen to nondimensionalize this group of equations. Results proved that nondimensionalized single particle theory equation groups obtained in this way included only one nondimensional parameter, which is one fewer than the G-W method's nondimensional Liouville light field equations, and that there were also only two nondimensional parameters that described the original conditions: nondimensional original intensity $P_{S0}$ and $\Delta\gamma_0/\gamma_0$. In this way, when $\Delta\gamma_0/\gamma_0$ is given, nondimensional saturation intensity and saturation length depend on just two nondimensional parameters. Based on these results, it is easy to use numerical calculation to approximately determine relatively simple scaling laws for the $P_{SS}$ and $z_{SS}$ of an untapered FEL amplifier with a certain wavelength. This article uses this method to give scaling laws for the $P_{SS}$ and $z_{SS}$ of millimeter wavelengths.

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[3] Lawrence Livermore National Laboratory

## 2. Groups of nondimensional parameters that determine nondimensional saturation intensity ($P_{SS}/P_{S0}$) and saturation length ($z_{SS}/\lambda_w$)

### 2.1 Nondimensionalization of fundamental equation groups

Single particle theory was used. At this point, the one-dimensional fundamental equation group for untapered wiggler free-electron lasers (ignoring the space charge effect) was the electron energy and phase equation from document [3] (here, the subscript $j$ indicates the $j$th electron)

$$\frac{d\gamma_j}{dz} = -\frac{\alpha_1 e B_w}{\gamma_j m c^2 k_w} e_s \sin\psi_j \ , \quad e_s = \frac{eE_s}{mc^2} \tag{1}$$

$$\frac{d\psi_j}{dz} = k_w - \frac{k_s}{2\gamma_j^2}\left(1 + \frac{\alpha_1 e^2 B_w}{m^2 c^4 k_w^2}\right) + \frac{\alpha_2 e B_w}{\gamma_j^2 m c^2 k_w} e_s \cos\psi_j + \frac{d\varphi}{dz} \tag{2}$$

$$\psi_j = (k_s + k_w) z - \omega_s t + \varphi + \psi_{0j}$$

The amplitude and phase equations of the laser electric field are

$$\frac{de_s}{dz} = \frac{2\pi e^2 B_w I}{m^2 c^5 k_w} \cdot \left\langle \frac{\sin\psi_j}{\gamma_j} \right\rangle , \quad I = n_0 e c \tag{3}$$

$$\frac{d\varphi}{dz} = \frac{2\pi e^2 B_w I}{m^2 c^5 k_w} \cdot \frac{1}{e_s} \cdot \left\langle \frac{\cos\psi_j}{\gamma_j} \right\rangle \tag{4}$$

In the above formulas, $e_s$ is the reduction amplitude of the laser electric field, $\phi$ is the phase of the light field, $k_w$ and $k_s$ are, respectively, the wave numbers of the wiggler and the fundamental mode laser, and $I$ is the current of the electron beam. For the circular polarization and linear polarization wigglers, $\alpha_1$ is, respectively, 1 and 0.5; $\alpha_2$ is, respectively, $\pm 1$ and 0.5.

Nondimensional electric field

$$\bar{e}_s = \frac{L_1^2 e B_w}{\gamma_0^2 m c^2} e_s \tag{5}$$

and nondimensional length

$$\bar{z} = \frac{z}{L_2} \tag{6}$$

are entered. Of these parameters, $L_1$ and $L_2$ are the constants of length dimension. At this point, equations (1) through (4) can be nondimensionalized to

$$\frac{d\gamma_j}{d\bar{z}} = -\frac{L_2\alpha_1\gamma_0^2}{L_1^2 k_w \gamma_j}\,\bar{e}_s \sin\psi_j \tag{1'}$$

$$\frac{d\psi_j}{d\bar{z}} = L_2 k_w - \frac{L_2}{2\gamma_j^2} k_s \left(1+\frac{\alpha_1 e^2 B_w^2}{m^2 c^4 k_w^2}\right) + \frac{L_2\alpha_2\gamma_0^2}{L_1^2 k_w \gamma_j^2}\,\bar{e}_s \cos\psi_j + \frac{d\varphi}{d\bar{z}} \tag{2'}$$

$$\frac{d\bar{e}_s}{d\bar{z}} = \frac{1}{c}\left(\frac{e}{mc^2}\right)^3 \frac{L_1^2 L_2 \lambda_w B_w^2 I}{\gamma_0^3} \left\langle \frac{\gamma_0}{\gamma_j}\sin\psi_j \right\rangle \tag{3'}$$

$$\frac{d\varphi}{d\bar{z}} = \frac{1}{c}\left(\frac{e}{mc^2}\right)^3 \frac{L_1^2 L_2 \lambda_w B_w^2 I}{\gamma_0^3}\,\frac{1}{\bar{e}_s} \left\langle \frac{\gamma_0}{\gamma_j}\cos\psi_j \right\rangle \tag{4'}$$

## 2.2 Groups of nondimensional parameters that determine $P_{SS}/P_{S0}$ and $z_{SS}/\lambda_w$

From formulas (1') through (4'), it is clear that when $L_1=L_2=\lambda_w$ and the following resonance relationship is used,

$$k_s\left(1+\frac{\alpha_1 e^2 B_w^2}{m^2 c^4 k_w^2}\right) = 2k_w\gamma_0^2 \tag{7}$$

and $\gamma_j$ in (1') through (4') appears in the form of $\gamma_j/\gamma_0$, (1') through (4') can therefore be written as

$$\frac{d(\gamma_j/\gamma_0)^2}{d\bar{z}} = -\frac{\alpha_1}{\pi}\,\bar{e}_s \sin\psi_j \tag{1''}$$

$$\frac{d\psi_j}{d\bar{z}} = 2\pi\left[1-\left(\frac{\gamma_0}{\gamma_j}\right)^2\right] + \frac{\alpha_2}{2\pi}\left(\frac{\gamma_0}{\gamma_j}\right)^2 \bar{e}_s\cos\psi_j + \frac{d\varphi}{d\bar{z}} \tag{2''}$$

$$\frac{d\bar{e}_s}{d\bar{z}} = \varepsilon\left\langle \frac{\gamma_0}{\gamma_j}\sin\psi_j \right\rangle \tag{3''}$$

$$\frac{d\varphi}{d\bar{z}} = \frac{\varepsilon}{\bar{e}_s}\left\langle \frac{\gamma_0}{\gamma_j}\cos\psi_j \right\rangle \tag{4''}$$

$$\gamma \equiv \frac{1}{c}\left(\frac{e}{mc^2}\right)^3 \frac{\lambda_w^4 B_w^2 I}{\gamma_0^3} \tag{8}$$

Equation groups (1") through (4") are equation groups of nondimensional variables $\gamma_j/\gamma_0$, $\psi_j$, $\bar{e}_s$, and $\phi$. They only contain one nondimensional coefficient, $\epsilon$, and no longer contain $\gamma_0$ alone. The original value of $\gamma_j/\gamma_0$ is determined by relativistic energy dispersion $\Delta\gamma_0/\gamma_0$;

the original value of $\psi_j$ is a constant that is unrelated to the parameters $I$, $\lambda_w$, $B_w$, $\lambda_s$, and $\gamma_0$. The original value of $\phi$ is zero, and the original value of $\bar{e}_s$ is $\bar{e}_{s0}$. The formula for $e_{s0}$ is

$$\bar{e}_{s0} = \frac{e}{mc^2} \frac{\lambda_w^2 B_w}{\gamma_0^2} e_{s0} \tag{9}$$

In the above formula, $e_{s0}$ is the original value of $e_s$.

The above deductions prove that the nondimensional variables $\gamma_j/\gamma_0$, $\psi_j$, $\bar{e}_s$, and $\phi$ are merely functions of $z$, $\epsilon$, $\Delta\gamma_0/\gamma_0$, and $(e/mc^2)(\lambda_w^2 B_w/\gamma_0^2)e_{s0}$. In particular, $\bar{e}_s(\bar{z})$ can be written as

$$\bar{e}_s(\bar{z}) = F(\bar{z}, \varepsilon, \frac{\Delta\gamma_0}{\gamma_0}, \frac{e}{mc^2} \frac{\lambda_w^2 B_w}{\gamma_0^2} e_{s0}), \quad \bar{z} = \frac{z}{\lambda_w} \tag{10}$$

Because nondimensional saturation length $\bar{z}_{SS}$ fulfills

$$\frac{\partial F}{\partial \bar{z}} \Big|_{\bar{z}=\bar{z}_{ss}} = 0$$

therefore, $\bar{z}_{SS}$ is just a function of $\phi$, $(e/mc^2)(\lambda_w^2 B_w/\gamma_0^2)e_{s0}$, and $\Delta\gamma/\gamma_0$. Note that

$$P_{ss} = \frac{m^2 c^5}{8\pi e^2} e_{ss}^2, \quad P_{s0} = \frac{m^2 c^5}{8\pi e^2} e_{s0}^2$$

$$\bar{e}_{ss} = \bar{e}_s(\bar{z}_{ss}) = F(\bar{z}_{ss}, \varphi, \frac{\Delta\gamma_0}{\gamma_0}, \frac{e}{mc^2} \frac{\lambda_w^2 B_w}{\gamma_0^2} e_{s0})$$

$$e_{ss}^2 = \frac{m^2 c^4}{e^2} \frac{\gamma_0^4}{\lambda_w^4 B_w^2} \bar{e}_{ss}^2$$

The following theorem has been proved.

Theorem 1: For untapered wiggler free-electron laser amplifiers,

$$P_{ss} = \frac{m^4 c^9}{8\pi e^4} \frac{\gamma_0^4}{\lambda_w^2 B_w^2} F_1(\varepsilon, \frac{e}{mc^2} \frac{\lambda_w^2 B_w}{\gamma_0^2} e_{s0}, \frac{\Delta\gamma_0}{\gamma_0}) \tag{11}$$

$$z_{ss} = \lambda_w F_2(\varepsilon, \frac{e}{mc^2} \frac{\lambda_w^2 B_w}{\gamma_0^2} e_{s0}, \frac{\Delta\gamma_0}{\gamma_0}) \tag{12}$$

In these formulas, $F_1$ and $F_2$ are functions of $\epsilon$, $(e/mc^2)(\lambda_w^2 B_w/\gamma_0^2)e_{s0}$, and $\Delta\gamma_0/\gamma_0$.

Theorem 1 stands up to arbitrary choice of $I$, $\lambda_w$, $B_w$, $\lambda_s$, and $\gamma_0$.

Theorem 2: For conditions where $\Delta\gamma_0 = 0$ and $P_{SS}$ has approximately no relation to $e_{s0}$ (the G-W scaling law was obtained under these two assumptions), by taking $F_1(\epsilon) \propto j^{4/3}$, the G-W scaling law is obtained.

Actually, at this time,

$$P_{ss} \propto \frac{\gamma_0^4}{\lambda_w^4 B_w^2} \left( \frac{\lambda_w^4 B_w^2 I}{\gamma_0^3} \right)^{4/3} \propto \lambda_w^{2/3} a_w^{2/3} I^{4/3} \tag{13}$$

The above formula is the G-W scaling law.

## 3. Scaling laws for the saturation intensity and saturation length of millimeter wavelength free-electron lasers

According to Theorem 1 in part 2.2, using multiple approximating functions $F_1$ within certain parameter ranges, for given $\Delta\gamma_0/\gamma_0$ and $P_{S0}$, $P_{SS}$ can be written as

$$P_{SS} = A \frac{\gamma_0^4}{\lambda_w^4 B_w^4} j^{\alpha} \left( \frac{\lambda_w^2 B_w}{\gamma_0^2} \right)^{\beta} \tag{14}$$

In this way, the numerical calculation results of three $I$, $\gamma_0$, $B_w$, and $\lambda_w$ groups with characteristic meaning (abbreviated to parameter groups) can be used to determine the constants $A$, $\alpha$, and ß in formula (14), and then the computational results of other parameter groups can be used to test the formula.

We used the above method on a millimeter wavelength parameter group range of $200 \leq I/\mathrm{A \cdot cm^{-2}} \leq 1000$, $3.5 \leq \gamma_0 \leq 11.775$, $2.75 \leq \gamma_w/\mathrm{cm} \leq 16.5$, $0.2168 \leq \lambda_s/\mathrm{cm} \leq 2.168$, and $1329 \leq B_w/\mathrm{Gs} \leq 1626$ with an Aurora-1 [*Shuguang*-1] parameter group ($I = 500\mathrm{A/cm^2}$, $\gamma_0 = 7.85$, $\lambda_w = 11\mathrm{cm}$, $B_w = 4065\mathrm{Gs}$, and $\lambda_s = 0.867\mathrm{cm}$) as its center, and used the calculated results of document [3] to derive the following scaling laws for saturation intensity and saturation length of untapered millimeter wavelength free-electron laser amplifiers. When $P_{S0} = 0.5\mathrm{MW/cm^2}$ and $\Delta\gamma_0/\gamma_0 = 0.03$,

$$P_{ss}/\mathrm{MW} = 3.672\times 10^{7}(\varepsilon-0.203)\left(\frac{(\lambda_w/\mathrm{cm})^2(B_w/\mathrm{Gs})}{\gamma_0^2}\right)^{-1.5} \tag{15}$$

$$z_{ss}/\mathrm{cm} = 39.4\,(\lambda_w/\mathrm{cm})(\varepsilon-0.203)\left(\frac{(\lambda_w/\mathrm{cm})^2(B_w/\mathrm{Gs})}{\gamma_0^2}\right)^{-0.21} \tag{16}$$

These two formulas conform well to the computational results of the 55 parameter groups within the above parameter group range. For the great majority, relative error was less than 16%, and for a small minority, it was less than 15%. The range of change in $\epsilon$ for these parameter groups was $0.492 \le \epsilon \le 16.7$. When $I$ was low and $\lambda_s$ was low, $\epsilon$ would be low, and at this time $B_w$ was also low. In formulas (15) and (16), $\epsilon-0.203$ was a correction made when $\epsilon$ was lower, while under general conditions, $(\epsilon-0.203)\approx\epsilon$. Substituting $(\lambda_w^2 B_w/\gamma_0^2)$ from formula (16) into formula (15) yields

$$P_{ss}/\mathrm{MW} = 1.473\times 10^{-4}\left(\frac{z_{ss}}{\lambda_w}\right)^{7.143}(\varepsilon-0.203)^{2.857} \tag{17}$$

From the above formula, it is clear that when $P_{S0}$ and $\Delta\gamma_0/\Delta_0$ are given, what determines $P_{SS}$ is $\epsilon$ and $z_{SS}/\lambda_w$, not $\epsilon$ and $z_{SS}$.

According to document [4], growth rate $G(L)$ is as follows under low gain conditions,

$$G(L) = \ln\frac{P_s(L)}{P_{s0}} = -\frac{\omega_b^2 a_w^2 k_w L^3}{8c^2\gamma_0^3}(J_0-J_1)^2\frac{d^2}{d\Phi_L}\frac{\sin^2\Phi_L}{d\Phi_L} \tag{18}$$

$$\Phi_L \equiv \left[k_s\left(1-\frac{c}{v_b}\right)+k_w\right]\frac{L}{2}\quad,\quad \omega_b^2 \equiv \frac{4\pi n_0 e^2}{m} \tag{19}$$

while the highest growth rate in high gain situations is

$$G(L) = \ln\frac{P_s(L)}{P_{s0}} = \sqrt{3}\,\sqrt[3]{2}\left(\frac{\omega_b^2 a_w^2 k_w L^3}{8c^2\gamma_0^3}\right)^{1/3} \tag{19}$$

Note that

$$g(L) \equiv \frac{\omega_b^2 a_w^2 k_w L^3}{8c^2\gamma_0^3} = \frac{1}{8}\,\frac{1}{c}\left(\frac{e}{mc^2}\right)^3\frac{\lambda_w B_w^2 I L^3}{\gamma_0^3}$$

but when $L=\lambda_w$,

$$g(\lambda_w) = \frac{1}{8}\,\frac{1}{c}\left(\frac{e}{mc^2}\right)^3\frac{\lambda_w^4 B_w^2 I}{\gamma_0^3} \tag{20}$$

As a result,

$$\varepsilon = 8g(\lambda_w) \tag{21}$$

Therefore, $\epsilon$ is a physical quantity that describes the growth rate where $z=\lambda_w$. In this way, according to the theorems in part 2, when $P_{S0}$ and $\Delta\gamma_0/\gamma_0$ are given, $P_{SS}$ is determined by and only determined by growth rate $j$ and $z_{SS}/\lambda_w$; the quantitative relationship of millimeter wavelengths is similar to formula (17). It is thus clear that nondimensional saturation length has a large influence on $P_{SS}$.

When current $I$ is expressed in units of A/cm$^2$ and other physical quantities are expressed in Gaussian units,

$$\varepsilon \equiv \frac{1}{c}\left(\frac{e}{mc^2}\right)^3 \frac{\lambda_w^4 B_w^2 I}{\gamma_0^3} = 0.2 \times 10^{-10} \frac{\lambda_w^4 B_w^2 I}{\gamma_0^3}$$

From formula (15), when $\epsilon >> 0.203$, the following simplified formula can yield $P_{SS}$

$$\begin{aligned} P_{SS} &= 0.7344 \times 10^{-3} \cdot \varepsilon \cdot \lambda_w^{-3} B_w^{-1.5} \gamma_0^3 = 0.7344 \times 10^{-3} \lambda_w B_w^{0.5} I \\ &= 0.7344\,(\lambda_w^4 B_w^2 I^4)^{1/4} \end{aligned} \tag{22}$$

But when $j$ is less than 0.5, the error from simplified formula (22) may be very large.

On the basis of the preceding theorems and formula (15), it is possible to find the dependence of $P_{SS}$ on $P_{S0}$

$$P_{SS} = 4.36677 \times 10^7 \cdot P_{S0}^{0.25} \cdot (\varepsilon - 0.203) \cdot \left(\frac{\lambda_w^2 B_w}{\gamma_0^2}\right)^{-1.5} \tag{23}$$

We calculated the above 55 parameter groups when $P_{S0}$=1MW/cm$^2$, and as a result, when $P_{S0}$=1MW/cm$^2$, $P_{SS}$ was 1 to 1.2 times ($2^{0.25}=1.789$) greater than when $P_{S0}$=0.5MW/cm$^2$. More than half [of the $P_{SS}$] were more than 1.1 times greater, and where current $I$ was less than 300A, [$P_{SS}$] was less than 1.05 times greater. Of course, when there are large variations in $P_{S0}$, the dependence of $P_{SS}$ on $(\lambda_w^2 B_w/\gamma_0)e_{s0}$ will change, and the dependence of $P_{SS}$ on $P_{S0}$ will thus change.

Below are given comparisons of groups of numerical calculation results of $P_{SS}$ and $z_{SS}$ of parameter groups with characteristic meaning and comparisons of calculation results of formulas (15) and (16), where $P_{S0}$=0.5MW/cm$^2$ and $\Delta\gamma_0/\gamma_0$=0.03. At the same time, these

numerical calculation results are used to explain several physics problems.

(1) $\gamma_0 = 7.85$, $\lambda_w = 11\text{cm}$, $B_w = 4065\text{Gs}$, $\lambda_s = 0.867\text{cm}$, (1)电流 $I$ 改变

| | | | | | |
|---|---|---|---|---|---|
| $I$ (A/cm²) | 1000 | 625 | 500 | 300 | 200 |
| $P_{ss}$ (MW/cm²) | 532.8 | 311.4 | 257.4 | 141.2 | 80.3 |
| $\frac{z_{ss}}{\lambda_w}$ | 3.271 | 3.681 | 4.004 | 4.5001 | 4.91 |
| $\varepsilon - 0.203$ | 9.8 | 6.049 | 4.796 | 2.796 | 1.796 |
| $P_{ss}^{(1)}$ | 504.61 | 311.49 | 246.95 | 143.99 | 92.51 |
| $(\frac{z_{ss}}{\lambda_w})^{(1)}$ | 3.326 | 3.774 | 3.98 | 4.548 | 5.036 |

Key: (1) Current $I$ varies

Of the above values, $P_{SS}$ and $z_{SS}/\lambda_w$ are calculated values from document [3], and $P_{SS}^{(1)}$ and $(z_{SS}/\lambda_w)^{(1)}$ are values given by formulas (15) and (16).

(2) $\gamma_0 = 7.85$, $\lambda_w = 11\text{cm}$, $B_w = 1646\text{Gs}$, $\lambda_s = 0.2168\text{cm}$, (1)电流 $I$ 改变

| | | | |
|---|---|---|---|
| $I$ (A/cm²) | 625 | 500 | 300 |
| $P_{ss}$ (MW/cm²) | 155.8 | 121.5 | 57.9 |
| $\frac{z_{ss}}{\lambda_w}$ | 7.637 | 8.182 | 9.82 |
| $\varepsilon - 0.203$ | 0.822 | 0.617 | 0.289 |
| $P_{ss}^{(1)}$ | 164.3 | 123.3 | 57.8 |
| $(\frac{z_{ss}}{\lambda_w})^{(1)}$ | 7.620 | 8.200 | 9.996 |

(3) $\gamma_0 = 7.85$, $\lambda_w = 5.5\text{cm}$, $B_w = 8129\text{Gs}$, $\lambda_s = 0.4335\text{cm}$, 电流 $I$ 改变

| | | | |
|---|---|---|---|
| $I$ (A/cm²) | 625 | 500 | 300 |
| $P_{ss}$ (MW/cm²) | 197.8 | 150.8 | 72.17 |
| $\frac{z_{ss}}{\lambda_w}$ | 6.28 | 6.636 | 7.909 |
| $\varepsilon - 0.203$ | 1.36 | 1.047 | 0.547 |
| $P_{ss}^{(1)}$ | 198.1 | 152.51 | 79.68 |
| $(\frac{z_{ss}}{\lambda_w})^{(1)}$ | 6.37 | 6.82 | 8.08 |

Key: (1) Current $I$ varies

The above three groups of results were [obtained] under conditions of different $\lambda_s$ and $\lambda_w$ when studying the dependence of $P_{SS}$ and $z_{SS}/\lambda_w$ on $\epsilon$. Clearly, when $\epsilon >> 0.203$, $P_{SS}$ is truly approximately proportional to $j$, and when $\epsilon$ is smaller, $P_{SS}$ is approximately proportional to $\epsilon - 0.203$. We also used data from these three groups to determine the degree of $\epsilon - 0.203$ in formula (16).

4. $I = 625 A/cm^2$, $\gamma_0 = 7.85$, $\lambda_s = 0.867 cm$, (1) $\lambda_w$ 改变

| | | | | | |
|---|---|---|---|---|---|
| $\lambda_w$ (cm) | 16.5 | 14 | 11 | 8 | 5.5 |
| $B_w$ (Gs) | 2148 | 2786 | 4065 | 6655 | 11820 |
| $P_{ss}$ ($MW/cm^2$) | 339.1 | 326.1 | 311.4 | 301.9 | 283.3 |
| $\frac{z_{ss}}{\lambda_w}$ | 3.242 | 3.394 | 3.681 | 4.183 | 4.729 |
| $\varepsilon - 0.203$ | 8.635 | 7.503 | 6.049 | 4.483 | 3.143 |
| $P_{ss}^{(1)}$ | 342.95 | 330.35 | 311.5 | 286.53 | 261.65 |
| $(\frac{z_{ss}}{\lambda_w})^{(1)}$ | 3.287 | 3.461 | 3.774 | 4.166 | 4.741 |

Key: (1) $\lambda_w$ varies

We used the preceding group of results to determine the degrees and coefficients of $\lambda_w^2 B_w/\gamma_0^2$ in formulas (15) and (16). From this group of results, one can see that when $I$, $\gamma_0$, and $\lambda_s$ do not change and $\lambda_w$ is reduced, although $B_w$ increases a great deal, $\epsilon$ still decreases a great deal, so that even if $z_{SS}/\lambda_w$ rises, $P_{SS}$ still falls.

(5) $I = 625 A/cm^2$, $\gamma_0 = 7.85$, (1) $\lambda_w$ 和 $\lambda_s$ 成比例缩小

| | | | |
|---|---|---|---|
| $\lambda_w$ (cm) | 11 | 5.5 | 2.75 |
| $B_w$ (Gs) | 4065 | 8129 | 16260 |
| $\lambda_s$ (cm) | 0.867 | 0.4335 | 0.2168 |
| $P_{ss}$ ($MW/cm^2$) | 311.4 | 197.8 | 87.9 |
| $\frac{z_{ss}}{\lambda_w}$ | 3.681 | 6.18 | 12.2 |
| $\varepsilon - 0.203$ | 6.049 | 1.36 | 0.188 |
| $P_{ss}^{(1)}$ | 311.5 | 198.2 | 77.35 |
| $(\frac{z_{ss}}{\lambda_w})^{(1)}$ | 3.77 | 6.37 | 12.38 |

Key: (1) $\lambda_w$ and $\lambda_s$ are reduced proportionately

The group of results above shows that when $I$ and $\gamma_0$ do not change and $\lambda_w$ and $\lambda_s$ are reduced proportionately, the results are similar to those from group 4: even though $B_w$ increases a great deal, $j$ decreases very much, so that even if $z_{SS}/\lambda_w$ increases, $P_{SS}$ still decreases by a large margin.

(6) $\gamma_0 = 3.92$, $\lambda_w = 2.75$cm, $B_w = 16260$Gs, $\lambda_s = 0.867$cm, (1)$I$ 改变

| | | | |
|---|---|---|---|
| $I$ (A/cm$^2$) | 625 | 500 | 300 |
| $P_{ss}$ (MW/cm$^2$) | 148.2 | 110.3 | 54.19 |
| $\frac{z_{ss}}{\lambda_w}$ | 4.546 | 4.729 | 5.454 |
| $\varepsilon - 0.203$ | 2.927 | 2.301 | 1.299 |
| $P_{ss}^{(1)}$ | 150.43 | 118.25 | 66.78 |
| $(\frac{z_{ss}}{\lambda_w})^{(1)}$ | 4.527 | 4.82 | 5.592 |

Key: (1) $I$ varies

The preceding is a miniaturized wiggler parameter group where $\lambda_s$ stays at 0.867 cm and $B_w$ is increased, while $\lambda_w$ and $\gamma_0$ are simultaneously decreased. Compared with the results from the first group, $P_{SS}$ decreased by a large margin, because although $z_{SS}/\lambda_w$ increased, $\epsilon - 0.203$ declined a great deal. Compared with the fifth parameter group, when $\gamma_0$ increased from 3.92 to 7.85, the $P_{SS}$ of $I=300$ increased from 54.19 to 87.9.

(7) $\gamma_0 = 11.775$, $\lambda_w = 11$cm, $B_w = 2910$Gs, $\lambda_s = 0.2168$cm, (1)$I$ 改变

| | | | |
|---|---|---|---|
| $I$ (A/cm$^2$) | 625 | 500 | 300 |
| $P_{ss}$ (MW/cm$^2$) | 208.5 | 136.9 | 81.84 |
| $\frac{z_{ss}}{\lambda_w}$ | 8.09 | 8.77 | 10.6 |
| $\varepsilon - 0.203$ | 0.746 | 0.556 | 0.253 |
| $P_{ss}^{(1)}$ | 214.01 | 159.56 | 72.5 |
| $(\frac{z_{ss}}{\lambda_w})^{(1)}$ | 8.219 | 8.870 | 10.89 |

Key: (1) $I$ varies

The preceding is a parameter group where $\gamma_0$ is increased while $B_w$ is decreased to reduce $\lambda_s$ even more. Compared with the results of the first group, this happened because $\epsilon$ fell

by a large margin. Although $z_{SS}/\lambda_w$ increased, $P_{SS}$ still declined a great deal.

Finally, we want to point out that from the computational results given above, it is possible to find many examples where $\epsilon$ is low, whereas $z_{SS}/\lambda_w$ is high, and thus $P_{SS}$ is [also] high. For example, compare group number 5, where $\lambda_w$=2.75cm, and group number 2, where $I$=300A/cm$^2$.

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